INFLUENCE OF NONUNIFORM SURFACE TEMPERATURE DISTRIBUTION ON LAMINAR BOUNDARY-LAYER STABILITY

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At present there is a sufficiently large number of studies (see, e.g., [1, 2]), theoretical as well as experimental, which consider the influence of surface cooling or heating on the stability of a laminar boundary layer and its transition to turbulence. Investigations were carried out to explain the effect of temperature ratio, i.e., the ratio of surface temperature to external flow temperature with their values kept constant. It is well known that for a gas, heating results in a decrease and cooling results in an increase in critical Reynolds number. The latter factor can be used to laminarize boundary layer. However, in practical applications an appreciable gradient in temperature along the surface is often found to exist. Approximate theoretical analysis shows that such temperature gradients significantly influence the characteristics of a stationary boundary layer: velocity and temperature profiles and so on. Hence it must be expected that nonuniform surface temperature distribution will appreciably affect flow stability. Such studies have not been carried out for gases except in [3] where the influence of a heated leading edge on boundary-layer stability was considered. It is shown below that for constant total heat flux the point of instability could move upstream as well as downstream depending on the surface temperature distribution. In view of this, it follows that by an appropriate selection of surface temperature distribution it is possible to obtain larger runs of laminar flow which is exceptionally important from the point of view of boundary-layer control, and, in particular, laminarization.

1. Problem Formulation.

The stability of a plane, subsonic boundary layer with nonuniform surface temperature is considered. The mathematical model consists of a system of equations including the continuity equation, Navier—Stokes equations, and energy equation neglecting viscous dissipation:

$$\begin{gathered}\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=0,\\ \rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}+\frac{2}{3}\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right],\\ \rho\frac{Dv}{Dt}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{2}{3}\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\right],\\ \rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right),\\ D/Dt=\partial/\partial t+u\partial/\partial x+v\partial/\partial y.\end{gathered}\tag{1.1}$$

It is assumed that the fluid obeys the equation of state for a perfect gas. The boundary conditions are:

$$\begin{gathered}u=0,\ T=T_w,\ v=0\ (y=0),\\ u\to u_e,\ T\to T_e\ (y\to\infty).\end{gathered}\tag{1.2a}$$

The surface temperature is specified by

$$T_w=T_e+T_1\xi^n.\tag{1.2b}$$

Here $\xi = x/L$; x, y are the streamwise and normal coordinates; u, v are the corresponding velocity components; p is the pressure; T, ρ are the fluid temperature and density; c_p is its specific heat at constant pressure; μ, λ are coefficients of viscosity and thermal conductivity; indices w and e denote values of parameters at the wall and in external flow respectively.

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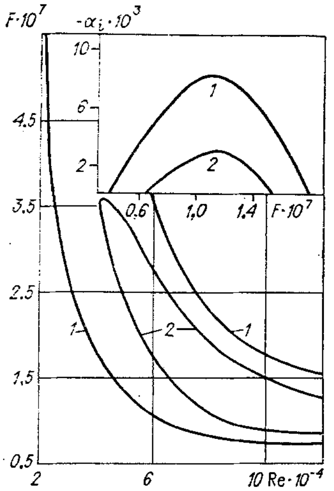

Fig. 1

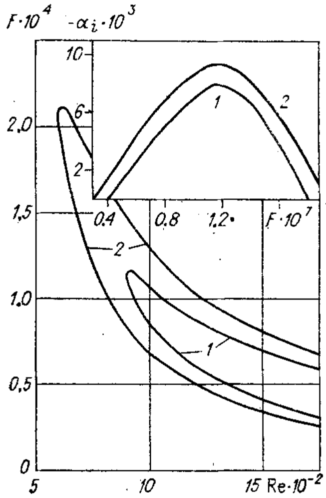

Fig. 2

The disturbance equations are obtained by linearizing the system of equations (1.1). Using the obvious assumption of small wavelength of disturbances λ_0 (Tollmien–Schlichting waves) compared to the characteristic length of the temperature variation which is of the same order as the surface length L, it is possible, with a parallel-flow approximation in the boundary layer [1, 2] to express the perturbation of stream function in the form of a plane wave $\varphi(y)\exp[i\alpha(x-ct)]$ where $\varphi(y)$ is the amplitude of the stream function.

To facilitate further analysis a new independent variable is introduced

$$\eta = \left(\frac{u_e}{x\nu_e}\right)^{1/2}\int_0^y \frac{\rho}{\rho_e}\,dy$$

(in further analysis a prime will denote differentiation with respect to this variable). It is known that in ξ, η coordinates, steady boundary-layer equations for given conditions (Faulkner–Skan type flows) permit similarity solutions.

The linearized equations can be reduced to one equation

$$\begin{aligned}(u-c)(\varphi''-\alpha^2\psi^2\varphi)-\varphi u''-\frac{\gamma}{i\alpha\,\mathrm{Re}}\left(\frac{\varphi^{\mathrm{IV}}}{\psi}-2\alpha^2\psi\varphi''+\alpha^4\psi^3\varphi\right)= \\ =\frac{1}{i\alpha\,\mathrm{Re}}\left\{\varphi^{\mathrm{III}}\left[2\left(\frac{\gamma}{\psi}\right)'-\chi\frac{\gamma}{\psi}\right]+\varphi''\left[\left(\frac{\gamma'}{\psi}\right)'-2\chi\left(\frac{\gamma}{\psi}\right)'-\frac{\gamma}{\psi}\chi'\right]-\right. \\ \left.-\alpha^2\varphi'(\gamma\psi'+2\psi\gamma')+\alpha^2\psi\varphi\left[\chi(\gamma\chi-\gamma')+\gamma''-\frac{\gamma}{\psi}\psi''\right]\right\}+\chi\varphi'(u-c)-\chi\varphi u',\end{aligned} \tag{1.3}$$

where α, c are the disturbance wave number and phase velocity; $\psi = T/T_e$; $\chi = \psi'/\psi$; $\gamma = \mu/\mu_e$; u is the free-stream velocity; Re is the Reynolds number based on displacement thickness. Since the boundary layer is assumed to be low subsonic, temperature fluctuation and changes in physical properties of the fluid are neglected [1]. The boundary conditions are:

$$\varphi = 0,\ \varphi' = 0\ (\eta = 0),\ \varphi \to 0,\ \varphi' \to 0\ (\eta \to \infty). \tag{1.4}$$

Equation (1.3) reduces to the well known Orr–Sommerfeld equation when $\psi = 1$.

Consequently, the present problem reduced to the determination of eigenvalues of Eq. (1.3) with homogeneous boundary conditions (1.4). In order to solve it, it is necessary to determine the coefficients of Eq. (1.3) which contain velocity, temperature, and viscosity distributions across the boundary layer, and also their derivatives in η.

Thus, it is necessary to know the mean flow characteristics which are determined from equations for a thermal boundary layer

$$\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=0,$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x},$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)$$

with boundary conditions

$$u=0,\ v=0,\ T=T_w=T_e+T_1\xi^n\ (y=0),$$
$$u\to u_e,\ T\to T_e\ (y\to\infty).$$

In similarity variables ξ, η, this boundary-value problem takes the form

$$\frac{\partial}{\partial\eta}(Kf'')+\frac{m+1}{2}ff''+m\left(\psi-f'^2\right)=0,$$
$$\frac{1}{\Pr}\frac{\partial}{\partial\eta}(K\Theta')+\frac{m+1}{2}f\Theta'+n(1-\Theta)f'=0; \quad (1.5)$$
$$f=0,\ f'=0,\ \Theta=0\ (\eta=0),$$
$$f'\to 1,\ \Theta\to 1\ (\eta\to\infty). \quad (1.6)$$

Here $K=\mu\rho/\mu_e\rho_e$; $m=\frac{\xi}{u_e}\frac{du_e}{d\xi}$; $f=\int\limits_0^\eta\frac{u}{u_e}d\eta$; $\Theta=\frac{T-T_w}{T_e-T_w}$; $\Pr=\mu c_p/\lambda$ is the Prandtl number.

2. Preliminary Qualitative Analysis.

Before obtaining a numerical solution to the boundary-value problems (1.5), (1.6) and (1.3), (1.4), some qualitative analysis is carried out for the nature of the influence of nonuniform surface temperature on boundary-layer stability. It is known that this influence is primarily introduced through the change in mean flow velocity profile.

Let us compare these profiles when the surface temperature is constant with those for power-law variation of surface temperature (1.2b). Consider the boundary layer on a flat plate ($m = 0$) with $K = 1$. It is convenient to return to the usual nondimensional coordinate $z=\eta+T_1\xi^n\int\limits_0^\eta(1-\Theta)\,d\eta$ and compare the values of z and z_0 for two different velocity profiles at a given location. Here $z = y/\delta_*$, and the index 0 denotes constant surface temperature.

It is necessary to know functions $\Theta(\eta)$ and $\Theta_0(\eta)$ for a comparison. Their form is determined under appropriate conditions: i.e., we require constant total heat flux along the plate length for different surface temperature distributions. As mentioned above, in this case it is possible to judge which distribution is the most desirable from the point of view of stability

$$\int\limits_0^1\left(\lambda\frac{\partial T}{\partial y}\right)_w d\xi=\int\limits_0^1\left(\lambda_0\frac{\partial T_0}{\partial y}\right)_w d\xi.$$

This condition determines the relation between T_{10} and T_1:

$$T_{10}=T_1B,\quad B=\frac{1}{1+2n}\frac{\Theta'_w}{\Theta'_{0w}}.$$

Introducing a new function

$$Q(\xi,\eta)=(1-\Theta)\xi^n-(1-\Theta_0)B,$$

we get the required difference in the form

$$z-z_0=T_1\int\limits_0^\eta Q\,d\eta,$$

where the function Q satisfies the boundary-value problem

$$Q''+\frac{\Pr}{2}fQ'-n\Pr Qf'=0,$$
$$Q_w=\xi^n-B\ (\eta=0),$$
$$Q\to 0\ (\eta\to\infty).$$

It follows from an analysis of its solution that for all $\eta\geq 0$ with $Q_w>0$, the function $Q>0$, and when $Q_w<0$, $Q<0$.

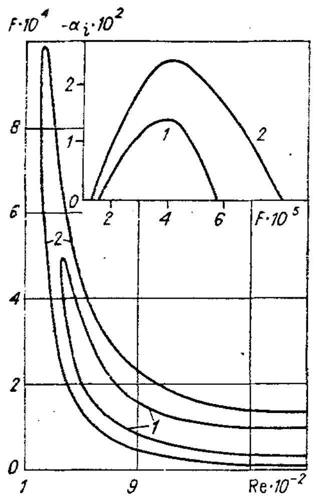

Fig. 3

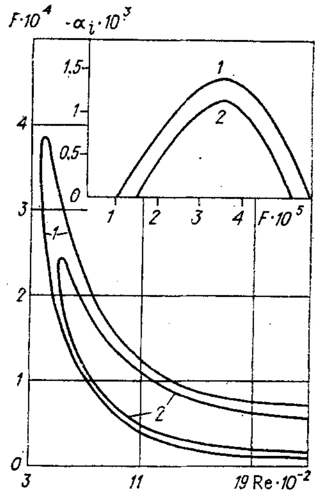

Fig. 4

Thus we arrive at the following conclusions on the basis of above analysis. With surface cooling ($T_1 < 0$) and heating ($T_1 > 0$) two effects of opposite nature, relative to the nature of the influence of nonuniform surface temperature distribution on boundary-layer stability, are possible, depending on the sign and value of its gradient: when $T_1 < 0$ and $n > 0$, it is stabilizing and when $n < 0$ it is destabilizing; when $T_1 > 0$ and $n > 0$, it is destabilizing and with $n < 0$ it is stabilizing.

3. Computational Results.

Results of computation of laminar boundary-layer stability to illustrate the above conclusions are given below. The eigenvalue problem (1.3), (1.4) was solved by the method of orthogonalization [4, 5]. The coefficients were determined from the solution of the boundary-value problem (1.5), (1.6).

Computed results are expressed in the form of neutral stability curves $F = F(\mathrm{Re})$, where $F = \omega v_e/u_e^2$, and relations for spatial-amplification factors $\alpha_i(F)$ at fixed Re. It was assumed that $K = 1$, $\mathrm{Pr} = 0.72$. In the case of surface cooling as well as heating, two surface temperature distributions were specified: linear ($n = 1$) and power law ($n = 0.4$). In all figures 1 denotes uniform surface temperature and 2 denotes nonuniform distribution.

Surface Cooling. 1. Figure 1 ($n = 1$, $T_1 = -0.5$, $\mathrm{Re} = 10^5$). It is seen that the region of unstable frequencies appreciably narrows. The minimum critical Reynolds number $\mathrm{Re}_* = 4 \cdot 10^4$ whereas for uniform cooling $\mathrm{Re}_* = 1.5 \cdot 10^3$. This means that the minimum length of the laminar segment increases by nearly three orders of magnitude. At lower values of F the difference is not that big but still quite appreciable. Thus, for $F = 2 \cdot 10^{-7}$, the length corresponding to the beginning of the growth of disturbances with nonuniform cooling increases nearly 2.5 times. Similar stabilizing influence is exhibited by nonuniform surface temperature distribution on the amplification factor α_i. It is seen that for the given Re it decreases nearly by a factor of three.

2. Figure 2 ($n = -0.4$, $T_1 = -0.1$, $\mathrm{Re} = 10^5$). Unlike the previous case, a destabilization and an increase in α_i are observed. The minimum value of $\mathrm{Re}_* \simeq 600$ as against $\mathrm{Re}_* \simeq 900$ with uniform cooling, which corresponds to approximately two-fold decrease in the length of the laminar segment.

Surface Heating. 1. Figure 3 ($n = 1$, $T_1 = 0.5$, $\mathrm{Re} = 2 \cdot 10^3$). There is a reduction in stability and an increase in the factor α_i; $\mathrm{Re}_* \simeq 200$ compared to $\mathrm{Re}_* \simeq 340$ for uniform heating. At lower values of the parameter F the difference is less significant. Where $\mathrm{Re} = 2 \cdot 10^3$, the quantity α_i increases by a factor of ~1.5.

2. Figure 4 ($n = -0.4$, $T_1 = 0.1$, $\mathrm{Re} = 2 \cdot 10^3$). Nonuniform surface heating is stabilizing and the amplification factor is reduced. The minimum length of the laminar segment increases by ~75%. When $F = 2 \cdot 10^{-4}$ this increase is not as significant and is ~23%, the value of α_i decreases by 25%.

Thus, the above analysis and computational results showed sufficiently large influence of nonuniform surface temperature distribution on subsonic laminar boundary layer. This influence is especially large for cooled surface if its temperature decreases downstream. Consequently, the application of such a temperature distribution is more effective for laminarization compared to uniformed distribution.

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EFFECT OF SUCTION ON LAMINAR COMPRESSIVE FLOW AND HEAT TRANSFER CLOSE TO A DISK ROTATING IN A GAS

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Suction for gas flowing over a wall may be used to combat flow instability in the region of the leading edge of a wing [1], and the nature of flow destabilization in which there is similar development of instability in the boundary layer on a rotating disk [2]. Suctioning of the boundary layer flowing over bodies or rotating surfaces is also an effective method for intensifying heat and mass transfer processes [3]. A knowledge of hydrodynamic and thermal characteristics of the boundary layer on a rotating disk is also necessary in a whole series of other technical situations [4].

In order to study uncompressed laminar flow close to a rotating disk with different external conditions, a method has been used successfully for averaging nonlinear inertial terms in equations of motion over the thickness of a boundary layer (the Slezkin—Targ method) making it possible to obtain analytical relationships for flow characteristics required in carrying out engineering calculations [5-8]. In the current work on the basis of a modified Slezkin—Targ method a study is made of a laminar boundary layer on an infinite disk rotating in a gas with presence of uniform suction from its surface taking account of medium compressibility. A process is considered for heat exchange between the disk and the external flow. Calculations are made for the thickness of hydrodynamic and thermal boundary layers, and also values of the coefficient of the disk resistance moment c_M and Nusselt number Nu in relation to suction parameter and the ratio of temperature in the external flow and in the disk. It is demonstrated that suction markedly affects the profile of hydrodynamic flow at the disk surface, increasing its resistance moment and heat emission. Results of calculating c_M are compared with known data for accurate solution of equations for a boundary layer on a rotating disk in the case of an incompressible liquid.

1. Ignoring viscous dissipation [5, 9] equations for spatial hydrodynamic and thermal boundary layers on a rotating disk in generally accepted notations are written in the form

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right); \tag{1.1}$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}\right)=\frac{\partial}{\partial z}\left(\eta\frac{\partial v}{\partial z}\right); \tag{1.2}$$

$$\frac{\partial}{\partial r}(\rho ru)+\frac{\partial}{\partial z}(\rho rw)=0; \tag{1.3}$$

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